Assignment 2

1. Prove that the sequence (a_n) defined recursively by $a_1 = 1$ and $a_{n+1} = a_n + \frac{1}{a_n}$, $n \geqslant 1$ is not bounded.

2. Prove that the sequence $a_n = (1 + \frac{1}{n})^n$, $n = 1, 2, \cdots$ is convergent in $\mathbb{R}$.

3. Show that a real sequence (a_n) is not bounded if and only if there is a subsequence (a_{n_k}) such that $|a_{n_k}| \geqslant k$ for all k.

4. Let (a_n) be a sequence in $\mathbb{R}$ which is bounded but not convergent. Show that there exist two subsequences converging to two different limits.

5. Prove that the following two conditions on sequence (a_n) in $\mathbb{R}$ are equivalent.
(a) (a_n) is bounded
(b) every subsequence of (a_n) has a convergent subsequence.

6. If (a_n) is a monotone sequence in $\mathbb{R}$ that has a bounded subsequence, show that (a_n) is convergent.